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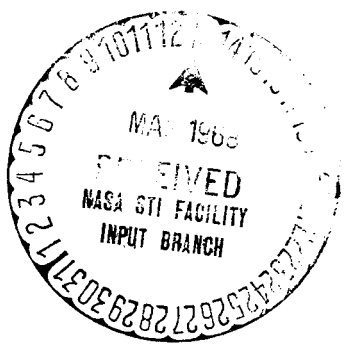

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INFRARED RADIATION METHOD OF DETERMINING THERMAL DIFFUSIVITY, HEAT CAPACITY AND THERMAL CONDUCTIVITY OF SOLID MATERIALS

Koichi Ogawa†

ABSTRACT: The paper presents a new and simple method of determining thermal diffusivity, heat capacity and thermal conductivity of solid materials, and some experimental results using the method. When the front surface of a plate specimen, of which the rear surface is thermally insulated, is heated by infrared radiation lamps under uniform and constant heat flow q(cal/cm^2·s), the rear surface temperature will be a quasi-steady state after a transient phenomenon, i.e., the temperature rises linearly with time. When heat is cut off suddenly under such a quasi-steady state, the rear surface temperature rises gradually from T(0) to a constant temperature T_M with time. Then "$t_{1/2}$", the time from heat cut-off to the time for $\{T_M-T(0)\}/2$ determines the thermal diffusivity α as

$$\alpha = 0.875 \frac{\delta^2}{\pi^2 t_{1/2}},$$

the heat capacity ρc is given by

$$\rho c = \frac{q t^*}{\delta T_M},$$

and the thermal conductivity k by

$$k = \alpha \cdot \rho c,$$

where δ is the thickness of the plate specimen and t is the heat duration time.*

These three thermal properties are determined for stainless steel, glass, bakelite, Japanese cypress and fiberglass-reinforced plastics at near 20° C.

It is shown that the thermal properties of non-metallic solid materials are simply determined by this new method and that their values are easily made available for practical applications.

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1. INTRODUCTION /1*

The thermal properties of new materials, such as fiberglass-reinforced plastics (FRP), that are being used for supersonic aircraft and rockets depend very strongly on the kind of glass and resin and the percentage content of glass. When these new materials are used as a part of the body of a plane, it becomes necessary to have a knowledge of the thermal properties of its materials. For example, when FRP is used in a supersonic aircraft or rocket, we need to know the thermal properties of FRP for a theoretical estimate of the temperature rise of the body due to aerodynamic heating. Since the thermal properties of new materials are a function of their use, we will discuss briefly in this paper a method for the determination of these thermal properties.

As a method of determining the thermal properties of solid materials by the use of non-stationary thermal conduction, Kawashita [1] and Yamaga [2] and others, for example, discuss such a method using a nichrome strip as the heat source, while W.J. Parker et al [3] used a flashlamp as the heat source. In the case of the former method, a very thin nichrome strip is sandwiched between two plates made from the material under consideration. An electric current is then passed through the strip to use the heat produced by the current as the source of heat. The temperatures of the front and rear surfaces of the specimen plate are measured by a thermocouple and the thermal properties of the specimen are determined from the non-stationary state of the temperature variation. On the other hand, in the case of the latter method the surface of a thin specimen is heated instantaneously by a special flashlamp. The thermal properties are determined by measuring the temperature change on the rear surface of the specimen by the use of the non-stationary state of the temperature variation.

In the present method, which uses an infrared lamp as the heat source, the surface of a specimen measuring 100 × 100 mm and a few mm thick is blackened by cobalt oxide in order to keep the incident rate of heat flow constant. By allowing the radiational heat of the infrared lamp to fall on the front surface and measuring the temperature variation on the rear surface with the thermocouple, the thermal properties of the material, i.e., thermal diffusivity α, thermal conductivity k, and heat capacity ρc are determined. /2

The advantages of the present method over those explained in the above are: (1) only one plate of the specimen is needed; (2) the apparatus is very simple; (3) thermal properties can be determined very simply from the temperature variation recorded; and (4) the time required for setting up the apparatus and for taking measurements is short, due to the simplicity of the apparatus. Furthermore, the accuracy of the result obtained is found (as shown in

* Numbers in the margin indicate pagination in the original text.

Table 1) to be satisfactory for practical purposes.

2. THEORY OF THE METHOD OF MEASUREMENT

As mentioned above, the thermal properties are to be determined by measuring the non-stationary temperature variation on the rear surface of a plate specimen when its surface is heated by the radiation from an infrared lamp. Since two types of non-stationary states (i.e., the type which appears immediately after the onset of heating and the type which appears after the heating is stopped), we will discuss in this paper two methods that make use of these two types of non-stationary states.

2.1. Basic Equation of Heat Conduction

If the flow of heat in the plate is only one-dimensional in the x-direction as shown in Figure 1, the equation of heat conduction is given by

$$\frac{\partial T(x, t)}{\partial t} = \alpha \frac{\partial^2 T(x, t)}{\partial x^2} \qquad (2.1)$$

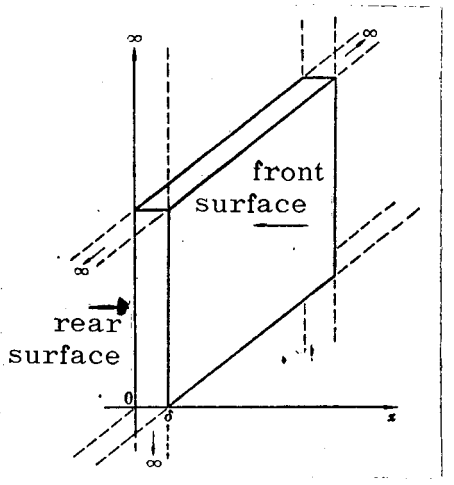

Fig. 1.

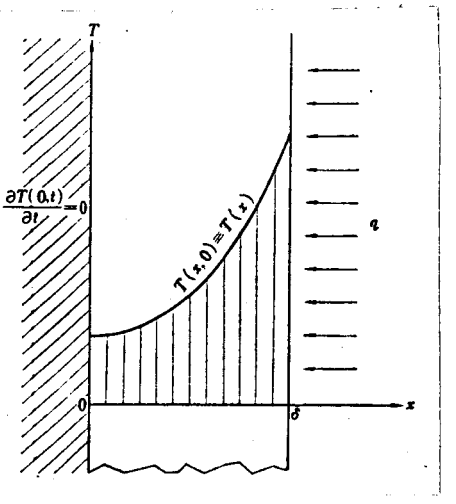

Fig. 2.

(a) Thermal conductivity in the case of heating the front surface with a heat flux q when the rear surface is insulated and the initial temperature distribution is zero. When the front surface ($x = \delta$) of the plate is heated by a spatially uniform and temporally constant radiant heat flux q, as shown in Figure 2, the boundary and initial conditions are given respectively by

$$\frac{\partial T(0, t)}{\partial x} = 0 \qquad \text{(boundary condition at } x = 0) \qquad (2.2)$$

$$k\,\frac{\partial T(\delta,t)}{\partial x} = -q \qquad \text{(boundary condition at } x = \delta\text{)} \tag{2.3}$$

$$T(x,0) = 0 \qquad \text{(initial condition)} \tag{2.4}$$

if we assume that the rear surface ($x = 0$) of the plate is insulated and the initial distribution of temperatures is zero. The solution of (2.1) which satisfies (2.2), (2.3) and (2.4) is given by [4,5]:

$$T(x,t)=\frac{qt}{\rho c\delta}+\frac{q\delta}{k}\left(\frac{3x^2-\delta^2}{6\delta^2}\right) -\frac{2q\delta}{k\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-\frac{n^2\pi^2\alpha}{\delta^2}t}\cos\frac{n\pi x}{\delta} \tag{2.5}$$

(b) Thermal conductivity of a plate when both front and rear surfaces are insulated and its initial distribution of temperature is $T(x)$. If we assume that there is no heat flux on the front surface ($x = \delta$) or on the rear surface ($x = 0$), i.e., when these surfaces are insulated, the boundary conditions become

$$\frac{\partial T(\delta,t)}{\partial x}=\frac{\partial T(0,t)}{\partial x}=0 \tag{2.6}$$

Furthermore, if the initial temperature distribution is given by $T(x)$ the initial condition becomes

$$T(x,0) = T(x). \tag{2.7}$$

The solution of (2.1) which satisfies (2.6) and (2.7) if given by [4]:

$$T(x,t) =\frac{1}{\delta}\int_0^{\delta}T(x)\,dx +\frac{2}{\delta}\sum_{n=1}^{\infty}e^{-\frac{n^2\pi^2\alpha}{\delta^2}t}\cos\frac{n\pi x}{\delta}\int_0^{\delta}T(x)\cos\frac{n\pi x}{\delta}dx \tag{2.8}$$

Here we have: /3

$$\alpha = \frac{k}{\rho c} : \text{ thermal diffusivity } (cm^2/s) \tag{2.9}$$

k : thermal conductivity (cal/cm·s·°C)
ρc : heat capacity (cal/cm^3·°C)
ρ : density (g/cm^3)
c : specific heat (cal/g·°C)
δ : thickness of the specimen (cm)
$T(x)$: initial temperature distribution (°C)

In part (a) above, we showed that the temperature variation inside a plate of infinite length and width δ, when its front surface is heated by a heat flux q as shown in Figure 1, is given by (2.5). From (2.5) it can be seen that long after the onset of the heating, the exponential terms become negligible and the temperature distribution within the plate is given by a quadratic function of x. If the heating is stopped when the temperature distribution is such a quadratic function, the temperature distribution inside the plate is a quadratic function of x so that the temperature variation after the stoppage of heating is given by (2.8) by taking the above quadratic function as the initial condition $T(x)$. Since a non-stationary state appears immediately after starting or stopping the heating, we will discuss in the following the thermal conduction after the onset and stoppage of the heating.

2.2. Thermal Conduction of a Plate After the Onset of the Heating

It was shown in the above that the thermal variation inside a plate whose rear surface is insulated and the front surface is heated by a heat flux q is given by (2.5). From this equation we can show that the temperature variations on the front ($x = \delta$) and rear ($x = 0$) surfaces are given respectively by

$$T(\delta, t) = \frac{qt}{\rho c \delta} + \frac{2q\delta}{6k} - \frac{2q\delta}{k\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\frac{n^2\pi^2\alpha}{\delta^2} t} \tag{2.10}$$

$$T(0, t) = \frac{qt}{qc\delta} - \frac{q\delta}{6k} - \frac{2q\delta}{k\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-\frac{n^2\pi^2\alpha}{\delta^2} t} \tag{2.11}$$

The temperature variations after the onset of heating on the front and rear surfaces are shown in Figure 3. After a sufficiently long time (which allows the neglect of the exponential terms) has elapsed, they are given by

$$T(\delta, t) = \frac{qt}{\rho c \delta} + \frac{2q\delta}{6k} \tag{2.12}$$

$$T(0, t) = \frac{qt}{\rho c \delta} - \frac{q\delta}{6k} \tag{2.13}$$

which show that the temperature variations on both front and rear surfaces have a linear rise with time.

Let us extend the linear portion of Figure 3 of the rear surface temperature given by (2.13) to the time axis and call the intersection t_0. From (2.13) we have

$$0 = \frac{qt_0}{\rho c \delta} - \frac{q\delta}{6k}$$

and the thermal conductivity k is given by

$$k = \frac{\rho c \delta^2}{6 t_0} \tag{2.14}$$

so that k can be determined by measuring the values of t_0 and ρc.

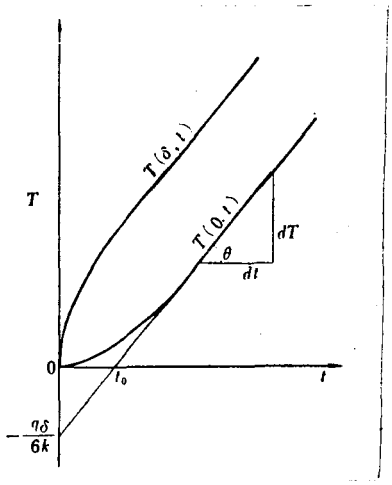

Fig. 3. Temperature Rise on the Rear Surface After the Onset of Heating.

By differentiating (2.13) with respect to t, we have

$$\frac{dT(0,t)}{dt} = \frac{q}{\rho c \delta}$$

from which the heat capacity ρc is given by

$$\rho c = \frac{q}{\delta \frac{dT}{dt}} . \tag{2.15}$$

Hence, ρc can be determined from the knowledge of $\frac{dT}{dt}$ of the quasi-stationary state. By substituting the value of ρc thus determined into (2.14) we can obtain the thermal conductivity k. Furthermore, the thermal diffusivity α can be determined from (2.9) by

$$a = \frac{k}{\rho c} \tag{2.16}$$

using the known values of ρc and k.

2.3. Thermal Conduction of a Plate After Stoppage of Heating

We have shown in the above that the temperature variation inside a plate is given by (2.5) when the plate is heated by a heat flux q and its rear surface is insulated. From (2.5) it is seen that the temperature distribution of a quasi-stationary state inside the plate, when it is heated sufficiently long to allow the neglect of the last-appearing exponential terms, becomes a quadratic function of x and that the shape of the temperature distribution does not change: it is simply shifted towards larger values when it is heated for longer times. If we adopt an arbitrary heating time t^* at which the exponential terms may be neglected, the temperature distribution inside the plate is given by

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$$T(x,t^*) = T(x) = \frac{q t^*}{\rho c \delta} + \frac{q \delta}{k} \frac{3x^2 - \delta^2}{6 \delta^2} \tag{2.17}$$

If we stop the infrared radiation after applying it for a time

t^* with a heat flux q, the temperature distribution within the plate at that moment is given by (2.17). We now use it as the initial condition for (2.8). The temperatures of the front and rear surfaces then $T(\delta)$, $T(0)$ are given respectively by

$$T(\delta)=\frac{qt^*}{\rho c\delta}+\frac{2q\delta}{6k} \tag{2.18}$$

$$T(0)=\frac{qt^*}{\rho c\delta}-\frac{q\delta}{6k} \tag{2.19}$$

These relations are shown schematically in Figure 4.

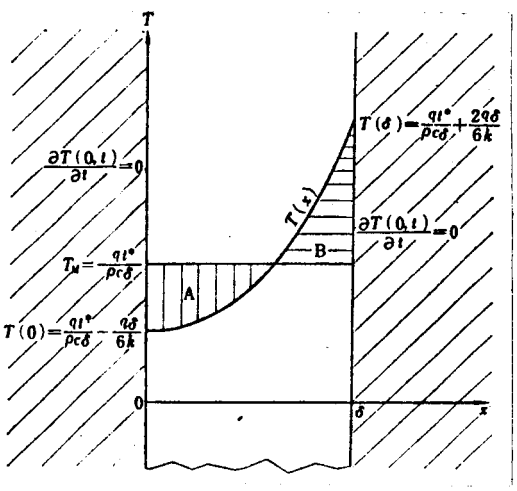

Fig. 4. Temperature Distribution at the Instant at Which the Heating of the Plate is Stopped.

By substituting the initial condition $T(x)$ given by (2.17) into the temperature distribution, (2.8) is obtained with the assumption of insulation at the front and rear surfaces; we then have:

$$\begin{aligned}T(x,t)=&\frac{1}{\delta}\int_0^\delta\left[\frac{qt^*}{\rho c\delta}+\frac{q\delta}{k}\left(\frac{3x^2-\delta^2}{6\delta^2}\right)\right]dx\\&+\frac{2}{\delta}\sum_{n=1}^{\infty}e^{-\frac{n^2\pi^2\alpha}{\delta^2}t}\cos\frac{n\pi x}{\delta}\int_0^\delta\left[\frac{qt^*}{qc\delta}\right.\\&\left.+\frac{q\delta}{k}\left(\frac{3x^2-\delta^2}{6\delta^2}\right)\right]\cos\frac{n\pi x}{\delta}dx\\=&\frac{qt^*}{qc\delta}+\frac{2q\delta}{k\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-\frac{n^2\pi^2\alpha}{\delta^2}t}\cos\frac{n\pi x}{\delta}\end{aligned} \tag{2.20}$$

In this equation, t stands for the time after the suspension of heating. By writing

$$\omega = \frac{\pi^2\alpha t}{\delta^2} \tag{2.21}$$

in (2.20) the temperature variations on the front and rear surfaces $T(\delta,t)$ and $T(0,t)$, respectively, become

$$T(\delta,t)=\frac{qt^*}{\rho c\delta}+\frac{2q\delta}{k\pi^2}\left(e^{-\omega}+\frac{e^{-2^2\omega}}{2^2}+\frac{e^{-3^2\omega}}{3^2}+\cdots\right) \tag{2.22}$$

$$T(0,t)=\frac{qt^*}{\rho c\delta}-\frac{2q\delta}{k\pi^2}\left(e^{-\omega}-\frac{e^{-2^2\omega}}{2^2}+\frac{e^{-3^2\omega}}{3^2}-\cdots\right) \tag{2.23}$$

If we let $\omega\to\infty$, that is, $t\to\infty$ in (2.22) and (2.23), we obtain

$$T(\delta,\infty) = T(0,\infty) = \frac{qt^*}{\rho c\delta}.$$

Calling this quantity T_M, i.e.,

$$T_M = \frac{qt^*}{\rho c\delta} \tag{2.24}$$

and substituting (2.18) and (2.19) into (2.22) and (2.23), respectively, we get

$$T(\delta, t)=T_M+\{T(\delta)-T_M\}\frac{6}{\pi^2}\left(e^{-\omega}+\frac{e^{-2^2\omega}}{2^2}+\frac{e^{-3^2\omega}}{3^2}+\cdots\right) \tag{2.25}$$

$$T(0, t)=T_M-\{T_M-T(0)\}\frac{12}{\pi^2}\left(e^{-\omega}-\frac{e^{-2^2\omega}}{2^2}+\frac{e^{-3^2\omega}}{3^2}-\cdots\right). \tag{2.26}$$

As can be seen from (2.18) and (2.19), the ratio of $\{T(\delta)-T_M\}$ and $\{T_M-T(0)\}$ becomes exactly 2:1. This shows that the ratio of the difference between the front surface temperature at the time of suspension of heating and the uniform temperature T_M which is reached after discontinuing heating to the corresponding difference between the rear surface temperature and the uniform temperature T_M is 2:1. The temperature distribution within the plate while it is being heated is given by (2.5), and the temperature distribution after suspension of heating is given by (2.20). Using these equations, the overall temperature change on the front and rear surfaces from the onset of heating to the uniform temperature after the stoppage of heating can be shown diagrammatically as in Figure 5. This also shows that with the initial temperature distribution $T(x)$ the regions A and B cancel each other and gradually come to a stationary state T_M. Transposing T_M from the right-hand side of (2.25) to the left-hand side, and dividing both sides by $\{T(\delta)-T_M\}$ we have T_δ^*, and similarly by adding a term $-T(0)$ to both sides of (2.26) and dividing both sides by $\{T_M-T(0)\}$ we have T_0^*. Then we will have:

$$T_\delta^*=\frac{T(\delta, t)-T_M}{T(\delta)-T_M}=\frac{6}{\pi^2}\left(e^{-\omega}+\frac{e^{-2^2\omega}}{2^2}+\frac{e^{-3^2\omega}}{3^2}+\cdots\right) \tag{2.27}$$ /5

$$T_0^*=\frac{T(0, t)-T(0)}{T_M-T(0)}=1-\frac{12}{\pi^2}\left(e^{-\omega}-\frac{e^{-2^2\omega}}{2^2}+\frac{e^{-3^2\omega}}{3^2}-\cdots\right) \tag{2.28}$$

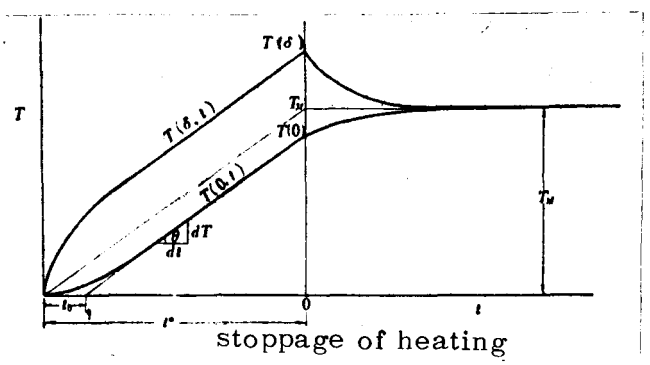

Fig. 5. Temperature Rise at Front and Rear Surfaces in a Heated Plane Specimen.

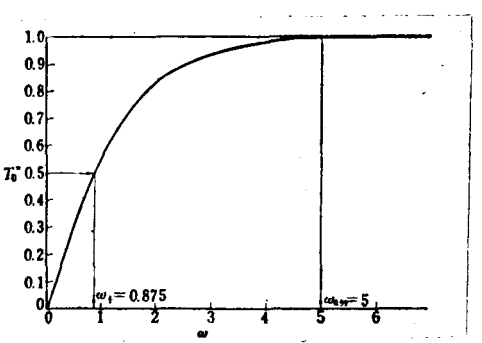

Fig. 6. Temperature Rise of the Rear Surface After Heating Stops.

Equations (2.27) and (2.28) represent the temperature variation which is made dimensionless at the front and rear surfaces after the stopping of heating, as shown in Figure 5.

In the case of measuring the front surface temperature $T(\delta,t)$ of a specimen which is heated by the radiation from a thermocouple, it is reported [9] that a slightly higher temperature than the actual temperature of the front surface is recorded when a thermocouple with a small heat capacity is used. In this report we will discuss a method of determining the thermal properties of the specimen by measuring the rear surface temperature $T(0,t)$ which is not directly affected by the heating radiation. The infinite sum on the right-hand side of (2.28) converges to $\pi^2/12$ when $\omega=0$ and becomes zero when $\omega\to\infty$ so we have $(T_0^*)_{\omega=0}=0$ and $(T_0^*)_{\omega\to\infty}=1$. Therefore, the relation between T_0^* and ω becomes as shown in Figure 6. If we determine the value of ω for which $T_0^*=0.5$ we find that $\omega=0.875$. If we call the time, which corresponds to this value of ω determined by (2.21), $t_{1/2}$, we have

$$\omega = 0.875 = \frac{\pi^2 \alpha t_{1/2}}{\delta^2}$$

The thermal diffusivity α is determined from this relation by

$$\alpha = 0.875 \frac{\delta^2}{\pi^2 t_{1/2}} \tag{2.29}$$

That is to say, when the front surface of a plate of thickness δ is heated with a heat flux q, heating is stopped after time t^*, and the plate is left standing, the rear surface temperature satisfies (2.28) and varies as shown in Figure 6 if the heat flux is assumed to be only one-dimensional in the direction of thickness. By measuring the time $t_{1/2}$ at which the temperature reaches half of the saturation temperature, and substituting its value into (2.29), we can easily determine the thermal diffusivity. In the above, we determined the value of ω at which $T_0^*=0.5$ is satisfied. However, we need not use the value of ω for which $T_0^*=0.5$ is valid. The reason for our using the value of ω which corresponds to $T_0^*=0.5$ is that the determination from the measured data of ω (or $t_{1/2}$) which satisfies $T_0^*=0.5$ is generally easier than the determination of ω for other values of T_0^*.

Next, when the front and rear surfaces of a plate are insulated and the front surface is heated with a heat flux q for a time t^*, the terminal temperature after stopping the heating becomes T_M, and (2.24) is valid. The heat capacity ρc can be determined from (2.24) by

$$\rho c = \frac{q t^*}{\delta T_M} \tag{2.30}$$

Further, the thermal diffusivity k can be determined from (2.9) by

$$k = \alpha \cdot \rho c \tag{2.31}$$

in terms of α and ρc determined by (2.29) and (2.30), respectively.

3. CHOICE OF THE HEATING TIME t^* WHICH IS NECESSARY FOR THE DETERMINATION OF THE INITIAL CONDITION $T(x)$ /6

As mentioned in *2.3*, the initial temperature distribution $T(x)$ is identified as the quadratic function of x given by (2.17), which is obtained after waiting for a sufficiently long time which permits neglect of the exponential terms of (2.5). It then becomes necessary to have some idea of the size of time t^* which justifies the neglect of the exponential terms and the approximation by a quadratic function. Since t^* is proportional to the square of the thickness δ of the plate and inversely proportional to the thermal diffusivity α, as will be shown later, the neglect of the exponential terms and the consequent approximation by a quadratic function $T(x)$ can be achieved for smaller thickness δ and larger thermal diffusivity α. Since, however, the thermal diffusivity α is unknown for an unknown specimen, it becomes necessary to determine the value approximately by the method described in *2.1*.

We first rewrite (2.5) as follows:

$$T(x,t)=\frac{qt}{\rho c\delta}+\frac{3q}{6k\delta}x^2-\frac{q\delta}{6k}\left(1+\frac{12}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}e^{-\omega n^2}\cos\frac{n\pi x}{\delta}\right) \tag{3.1}$$

where

$$\omega=\frac{\alpha\pi^2 t}{\delta^2} \tag{3.2}$$

If we consider the temperature variation at the rear surface ($x=0$) from (3.1), and if we call the expression in the brackets on the right-hand side of (3.1) y, we then have

$$y=1-\frac{12}{\pi^2}\left(e^{-\omega}-\frac{e^{-2^2\omega}}{2^2}+\frac{e^{-3^2\omega}}{3^2}-\cdots\right) \tag{3.3}$$

Equation (3.3) is exactly the same as the right-hand side of (2.28), and is shown graphically in Figure 6. If in (3.3) y becomes equal to unity, the initial condition $T(x)$ becomes exactly a quadratic function. The value of ω for which y approaches the value unity for 99%, i.e., the value of ω for which $y=0.99$, is found from Figure 5 to be $\omega\approx 5$. By calling the time corresponding to this, t^*, and substituting $\omega=5$ in (3.2), we obtain

$$t^* = \frac{5\delta^2}{\pi^2\alpha} \tag{3.4}$$

This means that by choosing heating time t^* such that $t^* \quad \frac{5\delta^2}{\pi^2\alpha}$ we have initial condition $T(x)$ which almost satisfies (2.17).

For example, let us determine t^* for three specimens: stainless steel, 3 mm thick; bakelite, 2 mm and 5 mm thick. Using the thermal diffusivity α [8] of stainless steel (0.0445 cm 2/s and the thermal diffusivity α [8] of bakelite (0.00111 cm^2/s), t^* for each

sample is determined from (3.4) as follows:

t^*	1.03 sec	>	stainless steel	$\delta = 0.3$ cm
t^*	18.3 sec	>	bakelite	$\delta = 0.2$ cm
t^*	115 sec	>	bakelite	$\delta = 0.5$ cm

As may be seen from the above, non-metallic specimens with small value of α and large thickness require a fairly large heating time. Therefore, in order to have accurate values of the heating time t^* and the time $t_{\frac{1}{2}}$ which is determined from the non-stationary state after the stoppage of heating, we have to use a higher chart speed. Since larger values of t^* mean a need for more recording paper (which is uneconomical), the appropriate thickness for non-metallic sample plates is about 1-2 mm, which corresponds (depending on the kind of specimen) to a heating time t^* of $\sim$20 seconds. On the other hand, a metal with a large value α has a large value ω, as seen from (3.2), and the non-stationary state disappears in a short time. Therefore, measurement of thermal properties can be made for metals, such as stainless steel, whose values of α are small, while the measurement by the present method becomes difficult for metals, such as copper, whose values of α are large.

4. MEASUREMENT OF HEAT FLUX q

The heat source which is used for heating specimens will be described in the next section. The radiant heat is generated by an infrared lamp. Here we will talk about the measurement of heat flux q which enters the specimen through radiational heating by an infrared lamp.

We have already shown that the temperature distribution within the plate specimen, whose rear surface is insulated to give an initial temperature distribution $T(x)$ and whose front surface is heated by a heat flux q, is given by (2.5). It was also shown that after a certain transition time, the exponential terms in (2.5) decrease rapidly with time so that the temperature distribution becomes that given by (2.17). If we differentiate the rear surface temperature $T(0,t^*)$, obtained from (2.17) with $x=0$, with respect to time t, we obtain

$$\frac{dT(0,t^*)}{dt} = \frac{q}{\rho c \delta} = \text{constant} \tag{4.1}$$

As shown in Figure 3, the temperature rises after a certain transition time, and its slope is seen from (4.1) to be proportional to the incident heat flux q. Hence, if we measure the change in the rear surface temperature of a metallic plate whose specific heat c, density ρ, and thickness δ are known, and find the rate of temperature rise dT/dt in a quasi-stationary state, the heat flux of the radiation that is to be taken as the standard is determined by [6,7]:

$$q = \rho c \delta \frac{dT}{dt} \tag{4.2}$$

5. EXPERIMENT AND DISCUSSION OF THE RESULTS

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5.1. Experiment

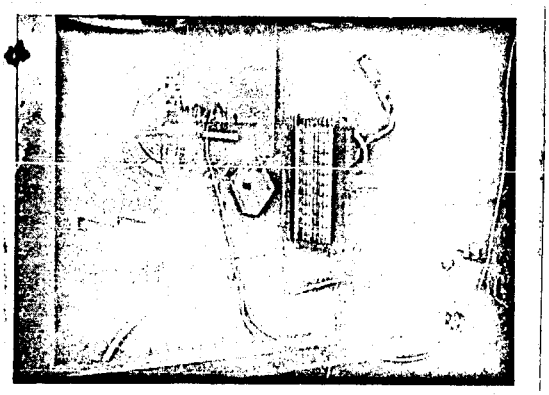
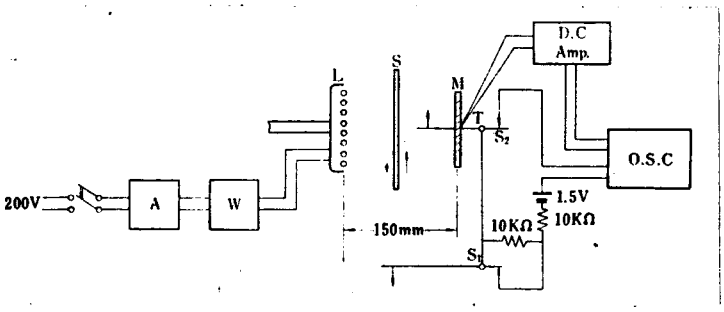


Fig. 7

We used as the heat source a heater composed of eight tubular infrared lamps placed an equal distance apart on the same plane, as shown in Figure 7. These lamps have a tube diameter of 10 mm, a filament length of 250 mm, and an output power of 1 kW per tube at 240 V and 3 kW at 480 V. As shown in Figure 8, the voltage to the lamps is applied through a slidac so as to be able to vary the output continuously between 0 and 24 kW. The value of the power applied to the heater is read directly



A: Slidac
S: Shutter
S : Marker for "Stop Heating"
W: Power Meter
M: Specimen
O.S.C.: Direct Recording Type Electromagnetic Oscillograph
L: Heater
S : Marker for "Start Heating"
D.C. Amp: Direct Current Amplifier
T: Thermocouple

Fig. 8. Experimental Apparatus

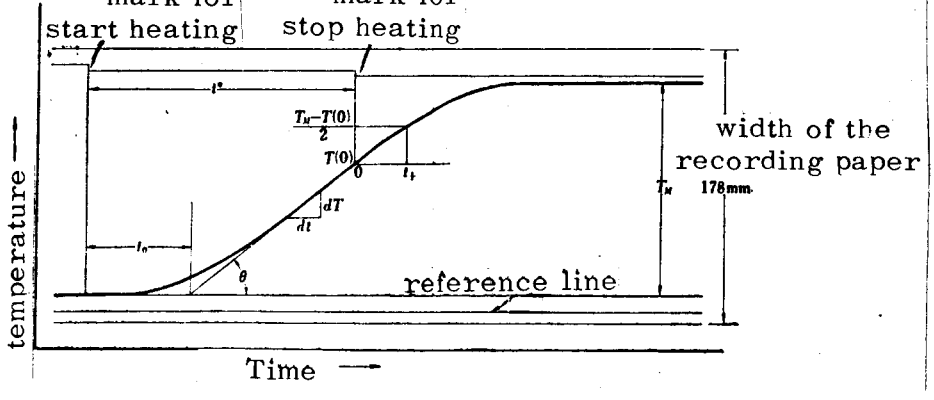


Fig. 9. Diagram Explaining the Recording of the Electromagnetic Oscillograph (Temperature Variation of the Rear Surface)

from a single-phase power meter. The heater and the specimen are kept a constant distance of 150 mm apart, and an aluminum plate 3 mm thick and measuring 330 mm x 330 mm is placed between the heater and the specimen as a shutter. At the time when heating is started, this shutter is opened instantaneously. As shown in Figures 8 and 9, when the shutter is half open to the heater, the marking for "start heating" is registered on the recording paper. Also, at the time when heating stops and the shutter covers half of the heater, the marking for "stop heating" is registered. The specimen is square, 100 mm x 100 mm.

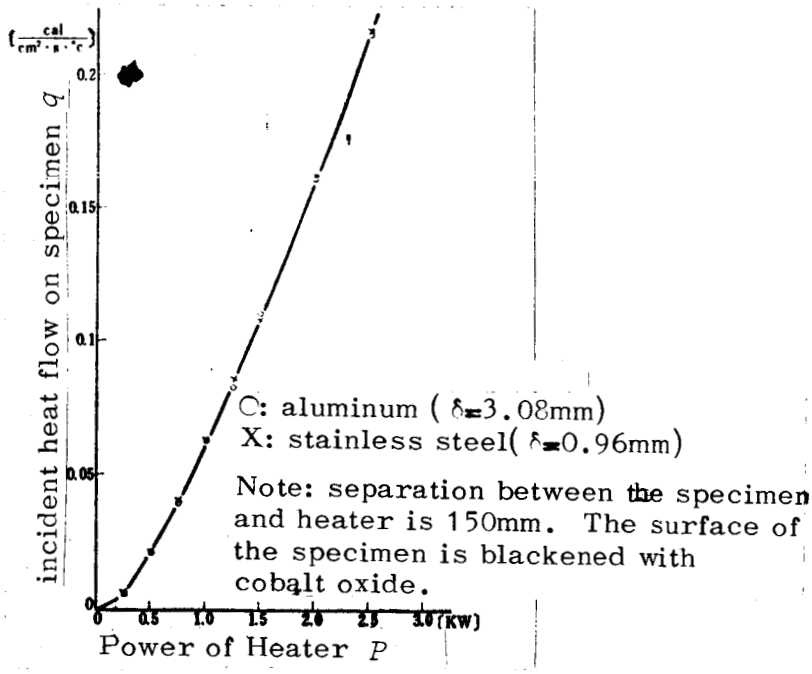


Fig. 10. Relationship Between the Incident Heat Flow on Specimen and the Power of the Heater

For the purpose of measuring the temperature variation of the rear surface, there is

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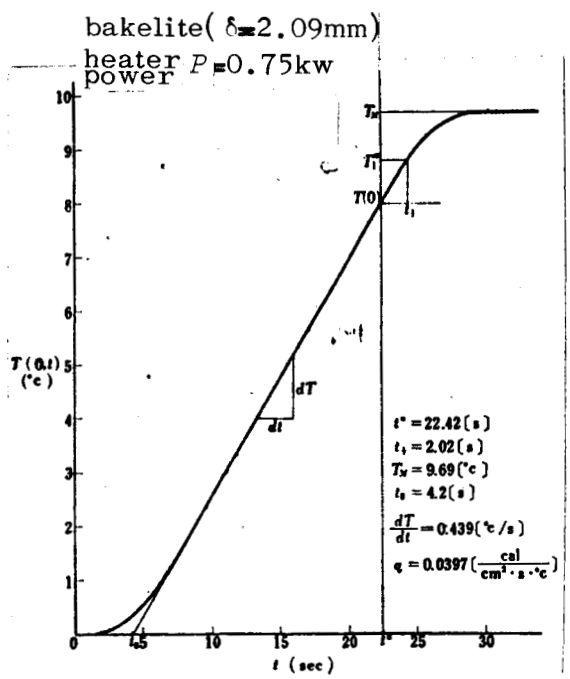

Fig. 11. Sample Recording [Bakelite]

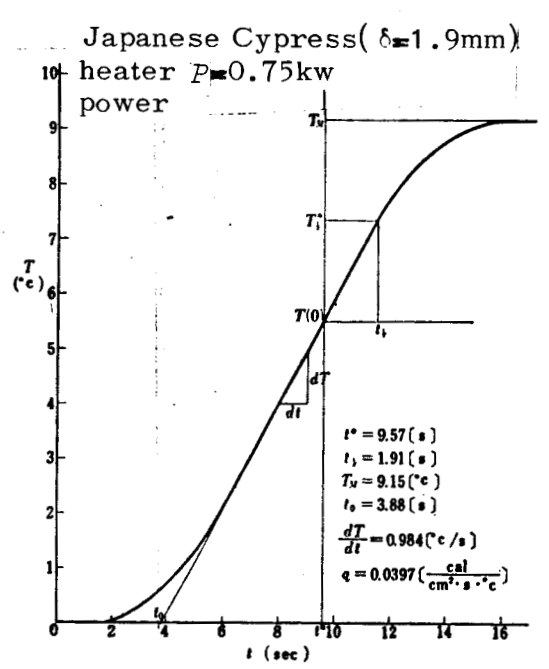

Fig. 12. Sample Recording [Japanese Cypress]

applied to the center of the rear surface an almel-chromel thermocouple of 0.1 ϕ with a pressure-type cement (Visca 18-M). Furthermore, to keep the heat flux q which enters the specimen through its front surface constant for different kinds of specimen, with a fixed distance between the heater and the sample and for fixed power of the heater, the front surface of the specimen is blackened by applying water-soluble cobalt oxide. The temperature variation of the rear surface is recorded with a direct-recording type electromagnetic oscillograph (Yokogawa Electric Works EMO-1, Vibrator G-100A), amplifying the minute output voltage of the thermocouple by two sets of direct-current amplifiers (Yokogawa Electric Works EM·A Type 31).

(1) Measurement of Heat Flux q Received by the Front Surface of the Specimen. As explained previously, the heat flux q is given by (4.2) in terms of the known values of ρ, c, and thickness δ. The relationship between the power p applied to the heater and the radiant heat flux q received by the specimen is shown in Figure 10 for stainless steel (ρ = 7.82 g/cm^3, c = 0.11 cal/g. °C, δ = 0.96 mm) [8] and aluminum (ρ = 2.7 g/cm^3, c = 0.215 cal/g. °C, δ = 3.08 mm) [8] for the case of a constant heater-specimen distance of 150 mm in the setup shown in Figure 8.

(2) Measurement of Thermal Properties. Keeping the power applied to the heater at a constant value of 1kW (it is necessary to choose an appropriate level for the power, depending on the thickness and properties of the specimen) the shutter is suddenly removed after the recording paper of the electromagnetic oscillograph has started to move (at the speed of 10 mm/s). When the shutter reveals one-half of the heater, as seen from Figure 8, it strikes the marker for "start heating" and a mark is registered on the recording paper. After heating for a fixed time t^*, the shutter closes. At this time, when the shutter closes and touches the marker for "stop heating" which is placed at the center of the heater, the other switch is released and the mark for "stop heating" is registered. These operations are shown in Figure 9. The width of the recording paper is 178 mm and it is adjusted by the gain of the direct current amplifier and the attenuator of the electromagnetic oscillograph to record a temperature variation of 10°C across the full width of the paper. For calibration of the recorded temperature, a portable direct-current potentiometer (Yokogawa Electric Works Type P-31A) is used. Examples of the experimental results obtained in this manner are shown in Figures 11-13. Table 1 also lists values of $t_{\frac{1}{2}}$, t^*, T_M, q (q = 0.0629 cal/cm^2·s for p = 1 kW), and values for eight kinds of material of heat capacity ρc, thermal conductivity k, and thermal diffusivity α. For the sake of comparison, the result obtained from (2.15), (2.14), and (2.9) and the values for four kinds of material that are reported in the literature [8] are also included in Table 1. For an easier visualization, the results thus obtained are arranged in a different setup in Table 2.

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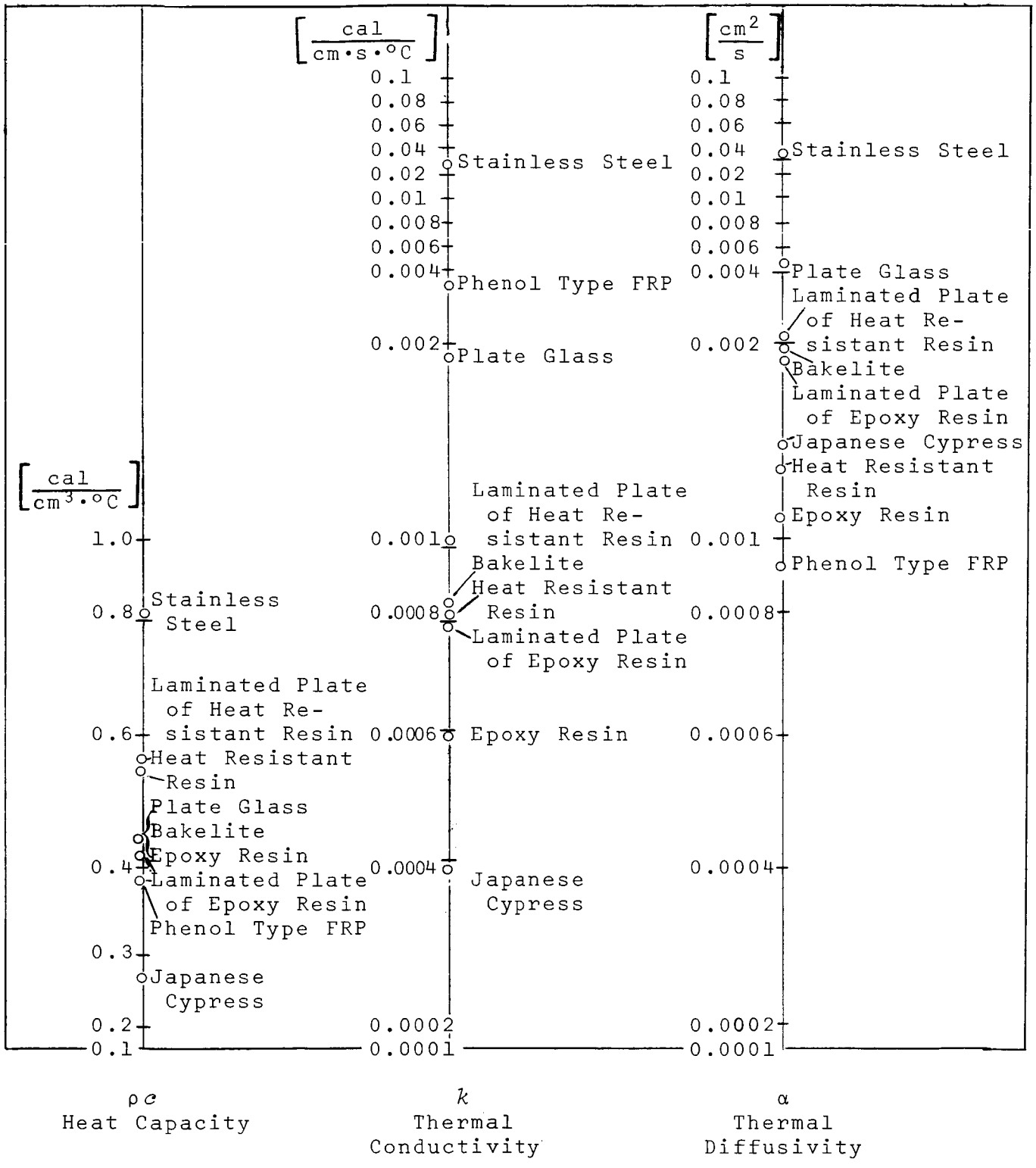
TABLE 1. RESULT OF MEASUREMENT OF CHARACTERISTIC VALUES

	Heat Capacity $\rho c \left[\frac{cal}{cm^3 \cdot °C}\right]$			Thermal Conductivity $k \left[\frac{cal}{cm \cdot s \cdot °C}\right]$			Thermal Diffusivity $\alpha \left[\frac{cm^2}{S}\right]$			Specific Weight $\rho \left[\frac{g}{cm^3}\right]$	
	(2.30)	(2.15)	Data from Heat Conduction Engineering	(2.31)	(2.14)	Data from Heat Conduction Engineering	(2.29)	(2.9)	Data from Heat Conduction Engineering		
				$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$	$\times 10^{-3}$		
Stainless Steel (5.18 mm)	0.834	0.830	0.86	33.61	30.93	38.9	40.32	37.27	44.5	7.8*	[illegible]
Plate Glass (1.3 mm)	0.438	0.433	0.483	1.850	2.170	1.810	4.22	4.94	3.34	2.7*	[illegible]
Bakelite Ø2.09 mm)	0.430	0.433	0.483	0.839	0.750	0.556	1.91	1.73	1.14	1.3*	[illegible]
Japanese Cypress (1.9 mm)	0.219	0.267	0.158	0.366	0.413	0.323	1.66	1.55	2.02	0.5*	[illegible]
Epoxy Resin (1.98 mm)	0.439	0.437	-	0.588	0.569	-	1.34	1.30	-	-	Epon 828
Laminated Plate of Epoxy Resin (1.38 mm)	0.405	0.404	-	0.761	0.686	-	1.88	1.70	-	1.9**	Laminated Plate Made from Epoxy Resin and Glass Cloth
Heat Resisting Resin (2.09 mm)	0.489	0.49	-	0.816	0.834	-	1.67	1.74	-	-	Mixture of Epoxy Resin (Epon 828 20, Hardening Agent 2.5) 1, Alum 4, Feldspar 4 and Special Heat-proof Glass 4
Laminated Plate of Heat Resisting Resin (2.2 mm)	0.498	0.492	-	1.035	1.005	-	2.07	2.04	-	-	Laminated Plate Made from Heat Resisting Resin and Glass Cloth
FRP of Phenol Type (2.97 mm)	0.375	0.364	-	2.440	0.884	-	0.917	0.243	-	1.9**	Phenol Resin Reinforced Plastic Having Glass Cloth as Its Foundation

* Data from Heat Conduction Engineering

** From Catalogue of Hitachi Synthetic Resin Laminated Plates

TABLE 2. RESULT OF MEASUREMENT OF THERMAL PROPERTIES



5.2. Discussion of the Results

The values of the thermal characteristics obtainable from the value of t_0 and the slope after the onset of heating and the similar values obtainable from T_M, $t_{\frac{1}{2}}$, and t^* determined after stopping heating are both included in Table 1. Contrary to our expectation that some discrepancy would exist between the values found in the literature [8] and those of the specimens tested here (because the specimens we used are not necessarily the same materials quoted in the literature), the comparison showed that they are very close. The method of determining the thermal properties [using (2.14), (2.15), and (2.9)] from the measured values of the slope and t_0 seems to be less accurate than the method of determining the thermal properties [using (2.29), (2.30), and (2.31)] from measurement of the values of T_M, $t_{\frac{1}{2}}$, and t^* because the former method contains larger errors in reading the data. This is because the value of t_0 is strongly dependent on the manner in which the slope of the temperature-time diagram is estimated. In contrast to this, in the latter case the reading of the value of t^* can be performed fairly accurately since there are markings for the starting and stopping of heating, and the values of $t_{\frac{1}{2}}$ and T_M can be read with considerable accuracy, as shown in Figure 9. This fact can also be recognized in Table 1 since the values determined [using (2.29), (2.30), and (2.31)] from the values of T_M, $t_{\frac{1}{2}}$, and t^* are closer to the values given in the literature.

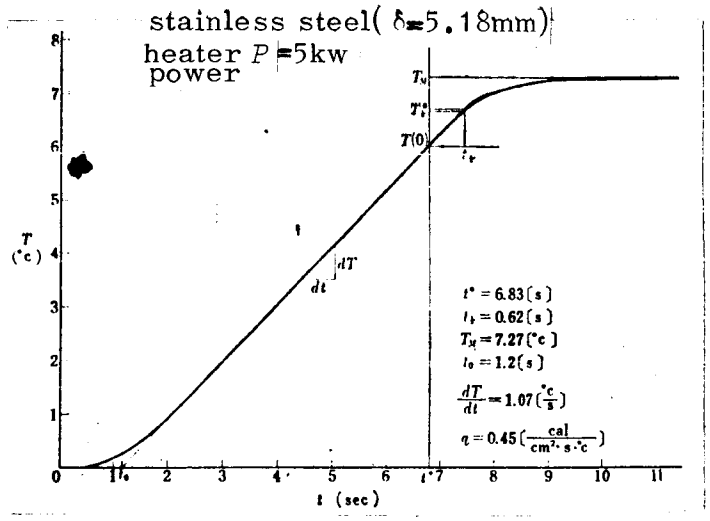


Fig. 13. Sample Recording [Stainless Steel]

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In addition there are several things that may be considered as possible causes of the errors. They are: (a) the measurement error of the incident heat flux on the specimen, (b) the error due to the assumption that the boundaries of the specimen are insulated, (c) the error due to the recording apparatus and introduced in the course of reading the recording, and (d) the error due to the manner in which the thermocouple is installed, etc. We will now take up the causes of these errors.

(a) Measurement Error in the Incident Heat Flux on the Specimen. In order to determine the thermal properties of unknown specimens based on the heat flux q on materials with known thermal properties, we require that these materials be exactly the same as the kinds of specimens quoted in the literature. If this is not

the case, it becomes necessary to know beforehand the exact values of the thermal properties of the materials that are going to be used as standards. In this paper, aluminum and stainless steel were used as standards to keep the error small.

(b) The Error Introduced by the Assumption that the Boundary Surfaces of the Specimen Are Insulated. Since the thermal properties of a specimen are such that they satisfy (2.1), that is, the solution for the case of insulated front and rear surfaces, it is not quite clear to what degree the required condition of insulation is actually met. To minimize the effect of this situation, in this experiment we kept the difference between the specimen temperature and the ambient temperature below 10°C. However, there is a small amount of heat transfer at the boundaries, so that a certain error due to this heat transfer is inevitable.

(c) Errors in Recording Apparatus and Reading. Since the /11
heater consists of eight infrared ray lamps which are parallel to each other, the heating rate at a point directly above a lamp is different from that at a point between the lamps. The effect on the error due to this fact must also be taken into account, as well as the heating rate at points more than 100 mm away from the heater. In addition, there are errors due to the degree of parallelism and the distance between the lamp and the specimen. In this experiment, no special consideration was given to this point except for checking the distance and parallelism with a yardstick each time. For the calibration of the recording apparatus we used a potentiometer (Yokogawa Electric Works Type P-31A). However, we recommend that a calibrator of higher accuracy be used for the calibration of the temperature range below 10°C for an almel-chromel thermocouple. Finally, as to the error of reading of recorded data, noise crept into the process of amplifying fairly strongly, the very weak output voltage of the thermocouple by means of a direct-current amplifier. We tried to eliminate these noises, but were unable to do so and the noises were consequently recorded on the electromagnetic oscillograph as lines about 2 mm wide. The error introduced in the process of reading the data is expected to be small. Nonetheless, it should also be counted as one of the various errors.

(d) Error due to the Manner in Which the Thermocouple is Installed. In the case of metals, we can solder the thermocouple directly to the specimen so that there is no error to speak of. As a method of measuring the rear surface temperature of materials such as non-metals to which a thermocouple cannot be soldered, we fastened the thermocouple to the surface using a pressure-type adhesive in this experiment. It is expected that there will be some effect produced by this method of installing the thermocouple. After the series of experiments reported in this paper was completed, we compared the cases of soldering and cementing by placing a plate of stainless steel almost at the same position. Little difference was observed between the two cases, and it was found that measurements can be made by installing the thermocouple either by soldering or

by cementing, up to a temperature of 180°C. Hence, we believe that the possible error introduced by the installation of the thermocouple with cement in measuring the rear surface temperature is small.

6. CONCLUSION

When the front surface of a plate specimen is heated by a heater (an infrared lamp) for a fixed time t^* and the temperature of the rear surface is measured, the non-stationary parts of the temperature variation of the rear surface appear immediately after the beginning of heating and after the heating is stopped. A method of determining the thermal properties of specimens and the actual experimental procedures were discussed. The non-stationary state, which appears after stopping of the heating and is actually measured in the experiment, coincides with the solution of the one-dimensional equation of heat conduction. In particular, we have discussed a method of determining the thermal diffusivity α from the time $t_{\frac{1}{2}}$ which is required for one-half of the difference $\left\{\frac{T_M - T(0)}{2}\right\}$ between the maximum value of the rear surface temperature T_M which is reached after stopping the heating and the rear surface temperature $T(0)$ at the time when the heating is stopped. We also have described a way of determining the heat capacity ρc from the maximum rear surface temperature T_M and the heating time t^*; and of determining the thermal conductivity k from ρc and α. Table 1 lists the results of determination of the thermal properties by this method for materials whose properties are known: stainless steel, plate glass, bakelite, and Japanese cypress. Also included in Table 1 are the thermal properties of materials whose properties are unknown: epoxy resin, a laminated plate of epoxy resin (using glass as the base), and a laminated plate of phenol resin (using glass as the base).

In the measurement of thermal properties by the present method, we need only a small amount of specimen which need only be placed in front of the heater. Hence, the whole apparatus is very simple. The thermal properties can be determined by substitution of the experimental data into simple formulas (2.29), (2.30), and (2.31). On the other hand, the heat flux q on the sample can be determined by the use of heat conduction of metals whose properties are well-known. However, there is a disadvantage in this method: it is not easy to apply to metals with high thermal conductivity because the method makes use of the non-stationary state of the temperature rise.

In conclusion we would like to thank Mr. Yukihiko Takenaka, Chief of the Thermal Elasticity Laboratory, Craft-Body Department, I, who gave us continual guidance; to Mr. Masayoshi Tamura, Group Leader, Instrumentation Section, Administration Department, who collaborated with us in the preparation of the specimens; and to Messrs. Tsunehiko Toyohara and Yoshio Noguchi, both of the Thermal Elasticity Laboratory, Craft-Body Department, I, for their cooperation in the experiments.

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